ABC: A Family of Fast Stream Ciphers

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Motivation

- Usage of new cryptographical primitives: T-functions and skew shifts.
- Illustration of our efficient techniques resting upon p-adic analysis and automata theory.
- A highly flexible framework for manufacturing fast and secure stream ciphers.
- Design simplicity.

T-functions: Definition, Part 1

- Every map G in $\mathbb{Z}/2^n\mathbb{Z}$

$$z \in \mathbb{Z}/2^n\mathbb{Z}, z = (z_0, \ldots, z_{n-1}) = \sum_{i=0}^{n-1} z_i 2^i,$$
$$G : \mathbb{Z}/2^n\mathbb{Z} \to \mathbb{Z}/2^n\mathbb{Z},$$
$$G : z \mapsto (\varphi_0(z_0, \ldots, z_{n-1}), \ldots, \varphi_{n-1}(z_0, \ldots, z_{n-1}))$$

can be given using n (boolean) coordinate functions

$$\varphi_i : (\mathbb{Z}/2\mathbb{Z})^n \to (\mathbb{Z}/2\mathbb{Z})^n, \ i = 0, \ldots, n.$$

T-functions: Definition, Part 2

- $G : \mathbb{Z}/2^n\mathbb{Z} \to \mathbb{Z}/2^n\mathbb{Z}, G = (\varphi_0, \ldots, \varphi_{n-1})$ is a **T-function** iff:

$$\begin{aligned}
\varphi_0 &= \varphi(z_0), \\
\varphi_1 &= \varphi(z_0, z_1), \\
&\ldots \\
\varphi_{n-2} &= \varphi(z_0, \ldots, z_{n-2}), \\
\varphi_{n-1} &= \varphi(z_0, \ldots, z_{n-2}, z_{n-1}).
\end{aligned}$$

- T-function = triangle function.
- A number of equivalent well-known definitions in mathematics.

T-functions: Invertibility

- The T-function $G = (\varphi_0, \ldots, \varphi_{n-1})$ in $\mathbb{Z}/2^n\mathbb{Z}$ is **invertible (bijective)** iff:
  - $\forall i = 0, \ldots, n-1$ the ANF of $\varphi_i(z_0, \ldots, z_i)$ is linear with respect to z_i.
- It is often advisable to use invertible functions as output functions in stream ciphers.

T-functions: Singe Cycle Property

- The T-function $G = (\varphi_0, \ldots, \varphi_{n-1})$ in $\mathbb{Z}/2^n\mathbb{Z}$ is **transitive (defines a singe-cycle permutation)** iff:
  - $\forall i = 0, \ldots, n-1$ the ANF of $\varphi_i(z_0, \ldots, z_i)$ is linear with respect to z_i,
  - the weight of φ_0 is 1,
  - for $i = 1, \ldots, n-1$ the weight of φ_i is odd,
- The linear complexity of the i-th coordinate sequence of a transitive function is $2^i + 1$. Single cycle functions guarantee the longest possible period and can be maps of choice for state update functions of stream ciphers.

T-functions: Examples of Transitive Functions, Part 1

- (Larin, early 80th; published 2002) A polynomial with integer coefficients is transitive $\pmod{2^n}$ iff it is transitive modulo 8.
- (Anashin, 1993) The function $F(x) = a_0 + b_1 \cdot (x \oplus a_1) + b_2 \cdot (x \oplus a_2) + \cdots$ is transitive $\pmod{2^n}$ iff it is transitive modulo 4.
- (Anashin, 1993) For arbitrary polynomials $u(x), v(x) \in \mathbb{Z}/2^n\mathbb{Z}[x]$ the integer-valued function

$$F(x) = \frac{v(x)}{2 \cdot u(x) + 1}$$

is transitive $\pmod{2^n}$ iff it is transitive modulo 8.

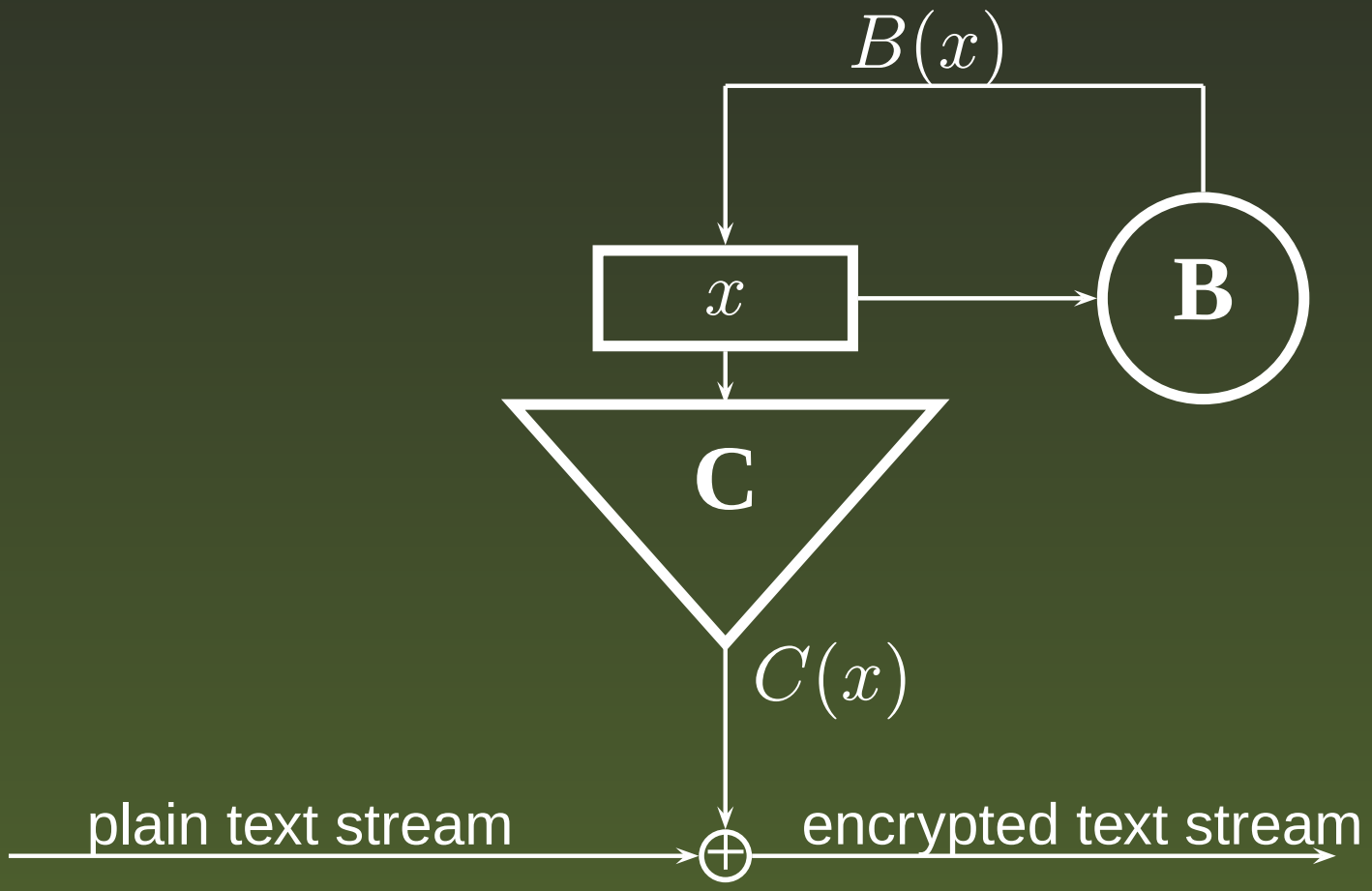
T-functions: Examples of Transitive Functions, Part 2

- (Kotomina, 1998) The function

  $$f(x) = (\ldots((((x+c_0)\oplus d_0)+c_1)\oplus d_1)+\cdots+c_m)\oplus d_m,$$

  is transitive $\pmod{2^n}$ iff f is transitive modulo 4
- (Anashin, 2002) The function $F(x) = a \cdot x + a^x$ is transitive $\pmod{2^n}$ iff a is odd.

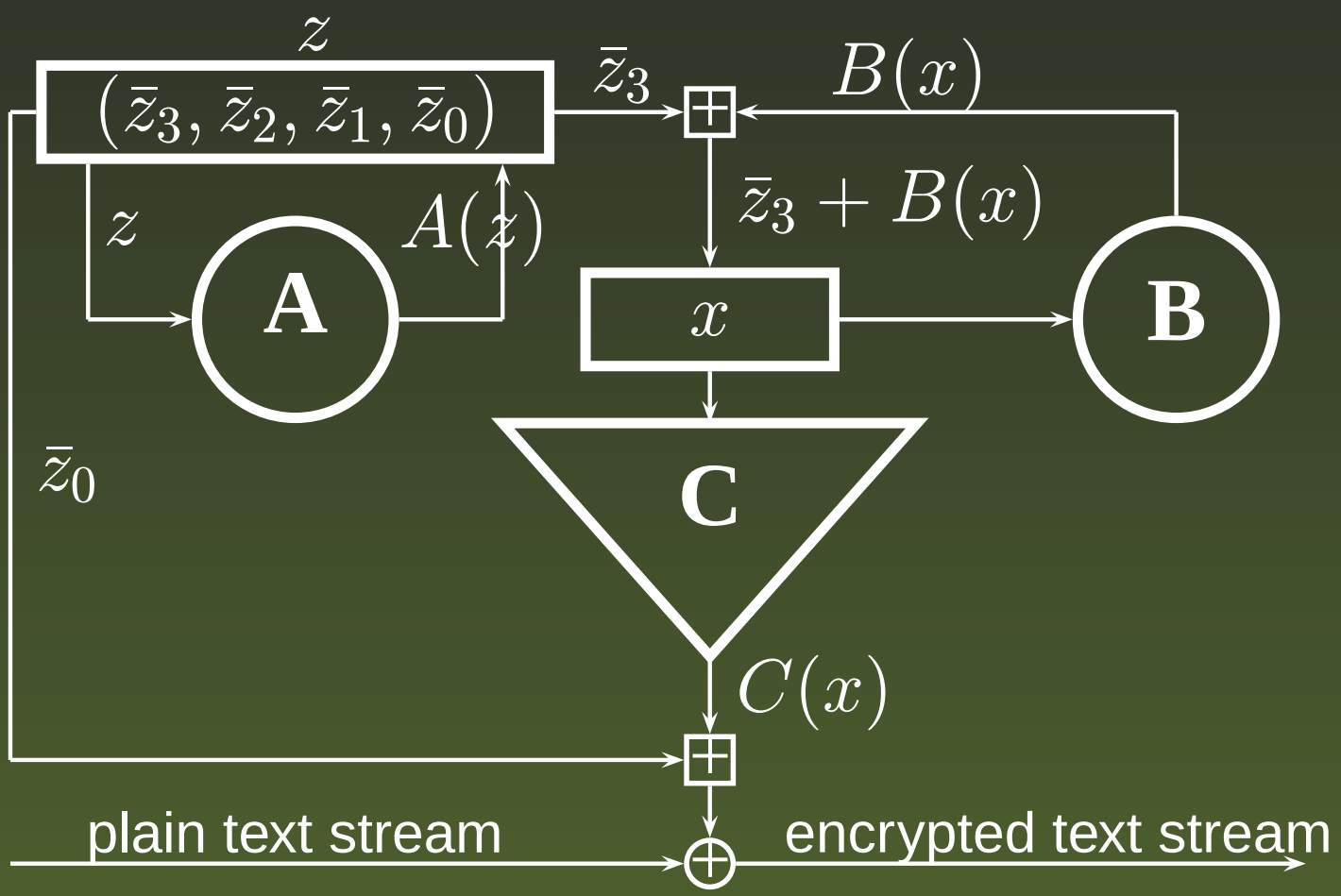
Traditional design of PRNG



B state transition function, period and distribution

C non-linear filter function, other crypto properties

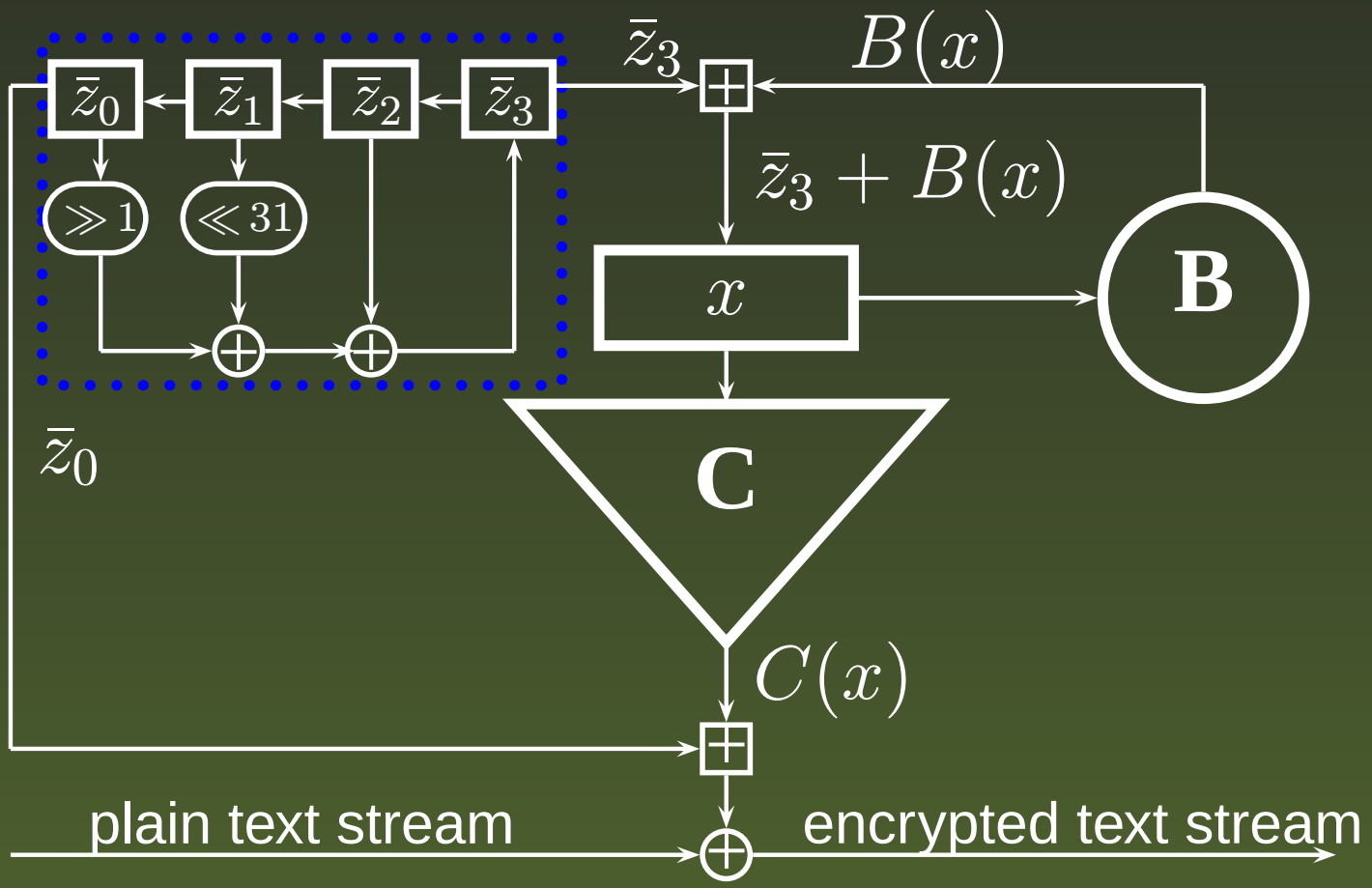
The ABC design pattern



$$\boxplus = + \pmod{2^{32}}$$

$$\oplus = \text{XOR}$$

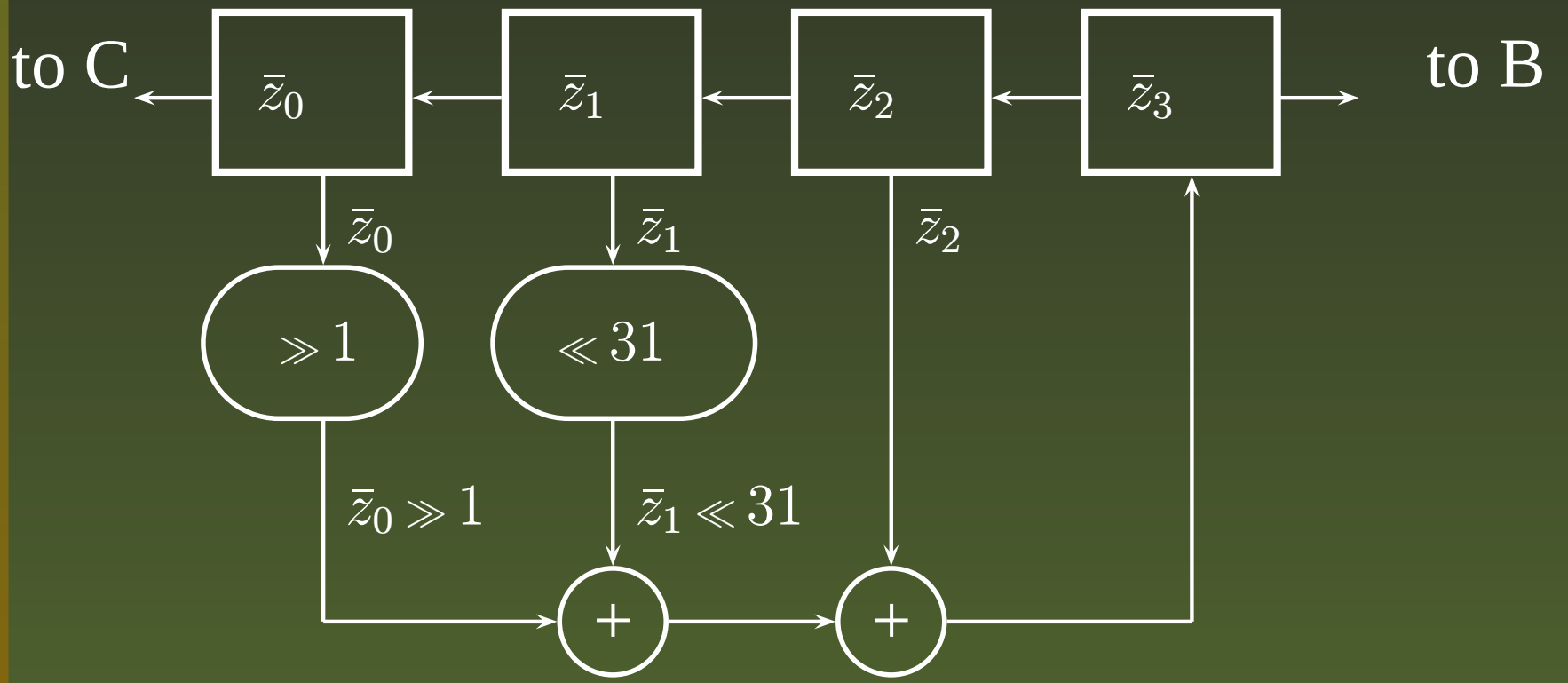
ABC: Function A



A : LFSR of period $2^{127} - 1$ for each 32-bit subword

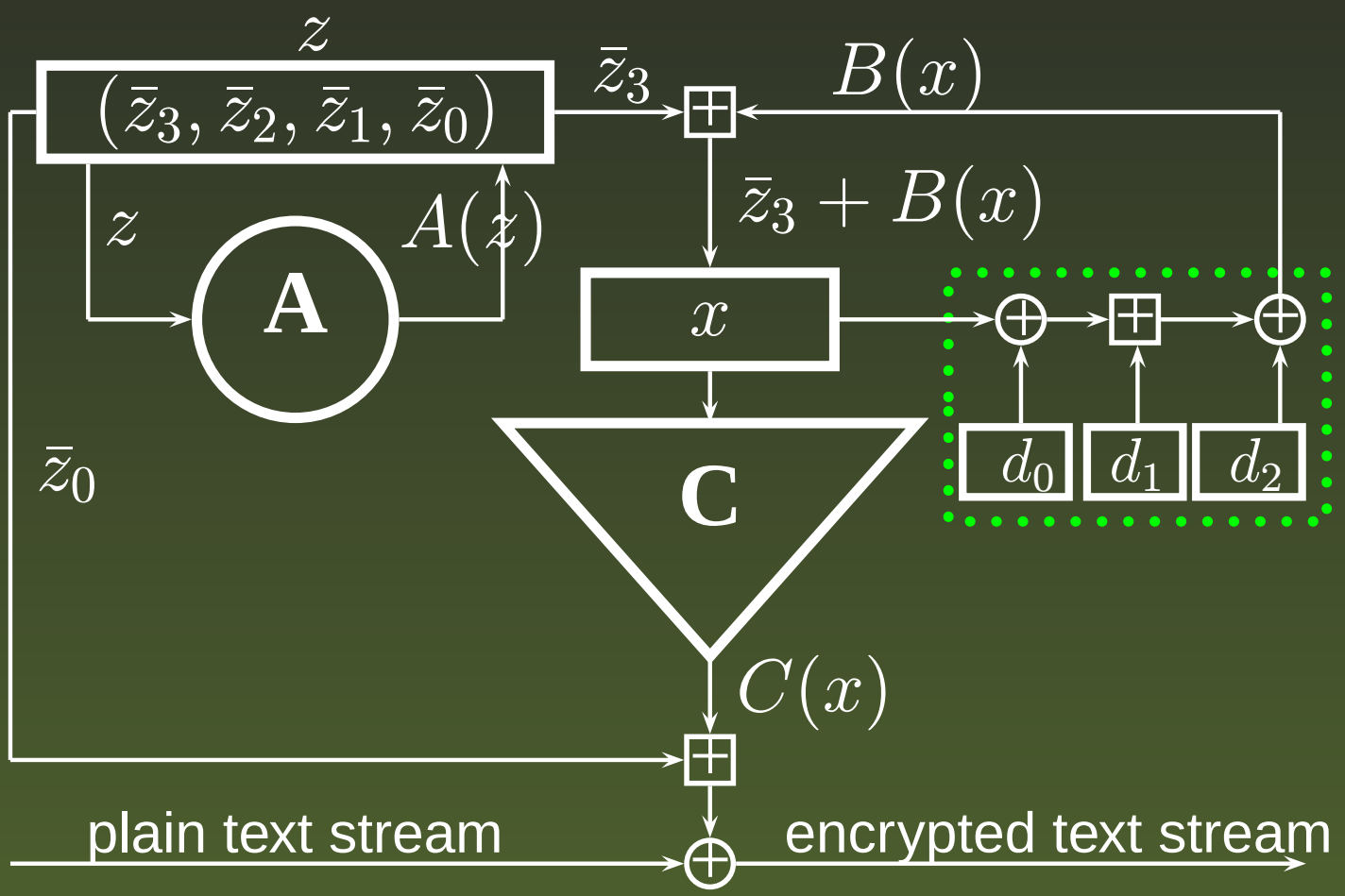
ABC: Function A in Detail

$$\phi(\theta) = (\theta^{127} + \theta^{63} + 1)\theta$$



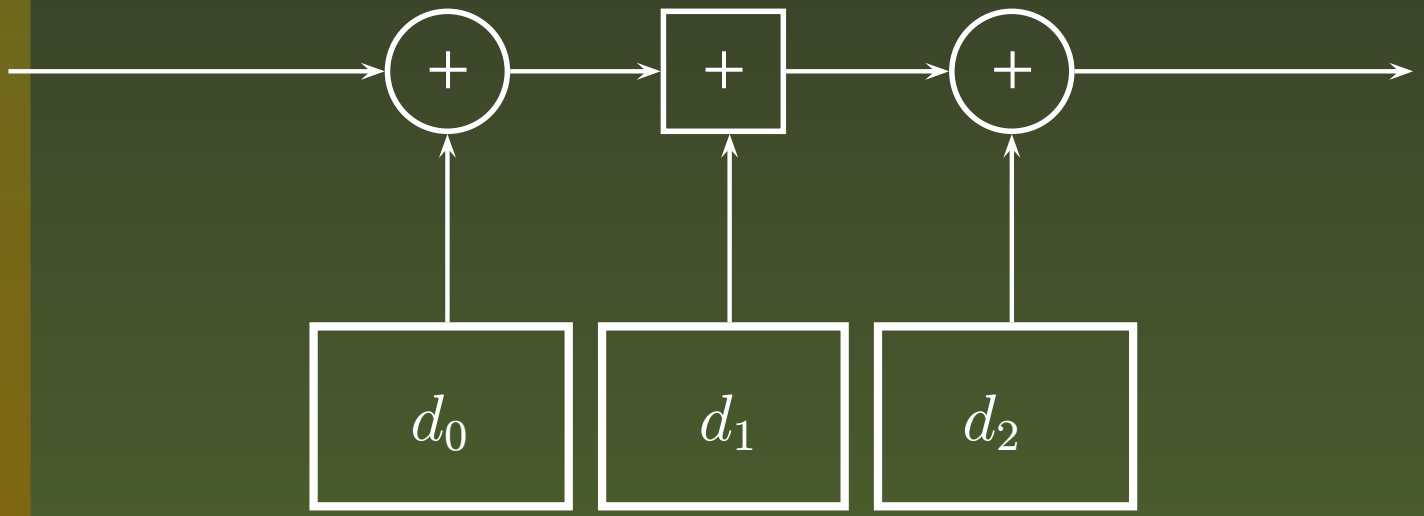
A : Word oriented computation of LFSR

ABC: Function B



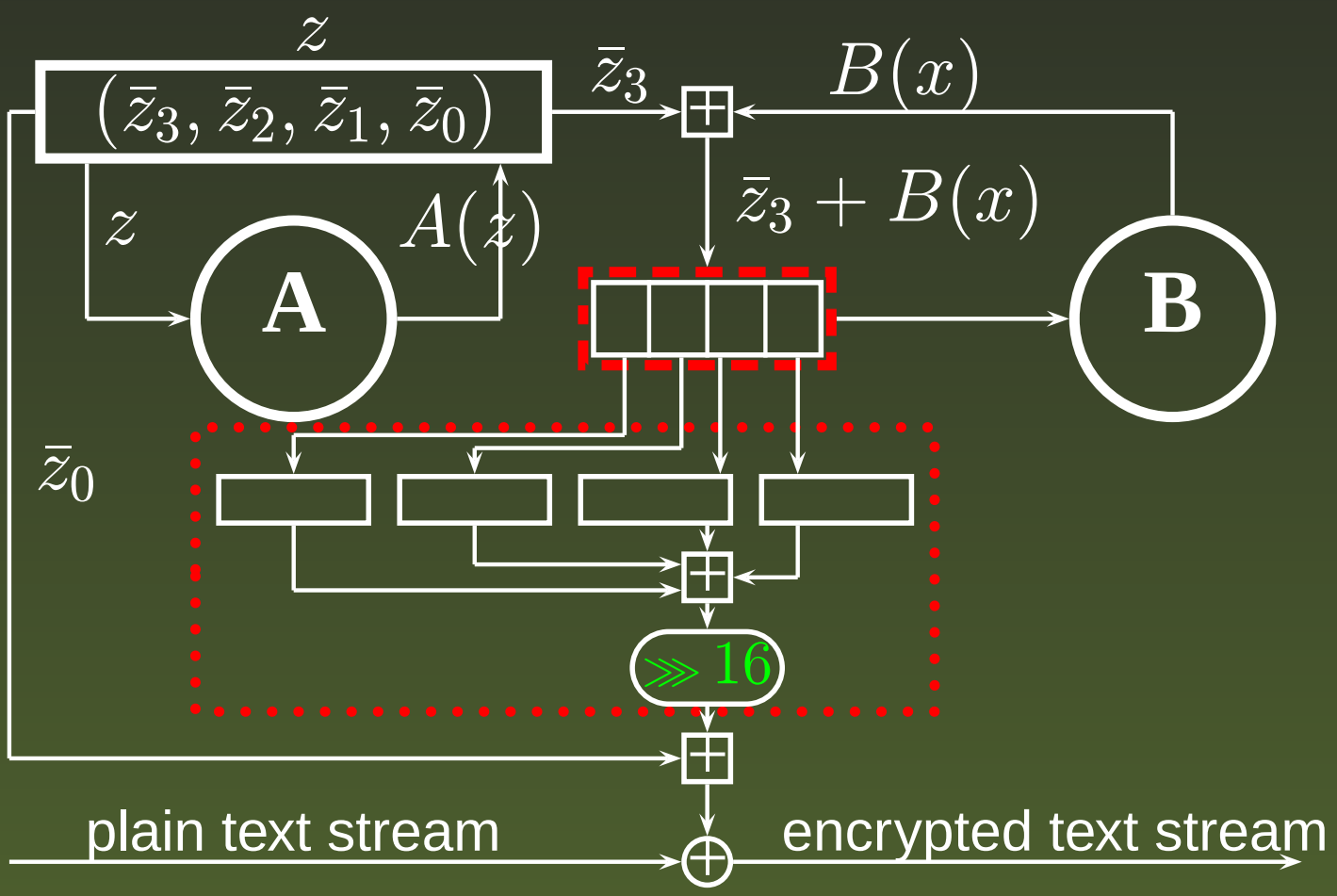
B : Defines a single cycle permutation over $\mathbb{Z}/2^{32}\mathbb{Z}$

ABC: Function B in Detail



$$B(x) = ((x \oplus d_0) + d_1) \oplus d_2 \pmod{2^{32}}$$

ABC: Function C

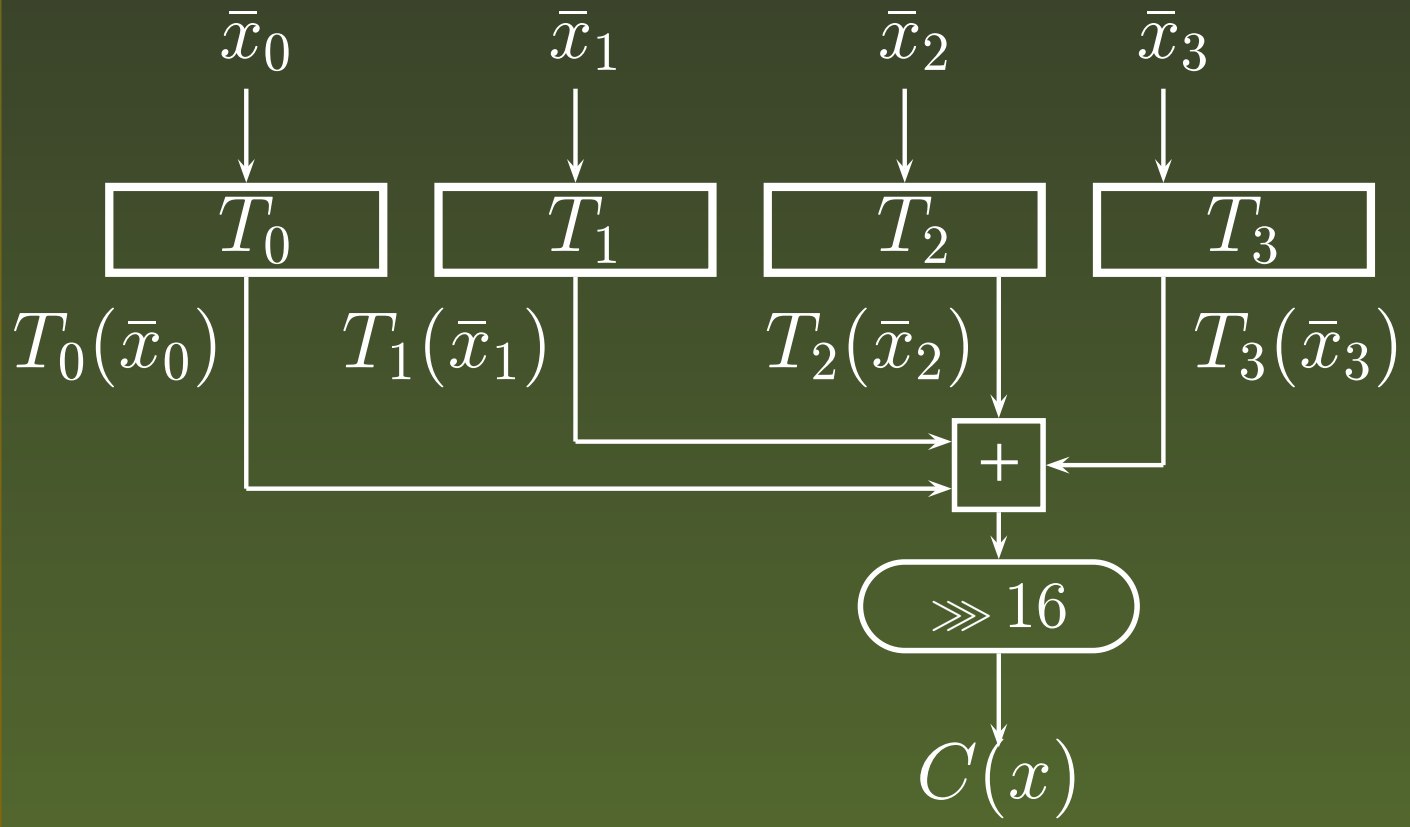


ABC: Function C in Detail

- $S(x) = e + \sum_{i=0}^{31} e_i \delta_i(x) \pmod{2^{32}}$, where
  - $\delta_i(x) \in \{0, 1\}$ = the i-th bit of x,
  - $e,\ e_i \in \mathbb{Z}/2^{32}\mathbb{Z}$,
  - $e_{31} \equiv 2^{16} \pmod{2^{17}}$.
- $C(x) = S(x) >>> 16 \pmod{2^{32}}$.

ABC: Function C in Detail

$$S(x) = e + \sum_{i=0}^{7} e_i \delta_i(x) + \cdots + \sum_{i=24}^{31} e_i \delta_i(x) \pmod{2^{32}}$$

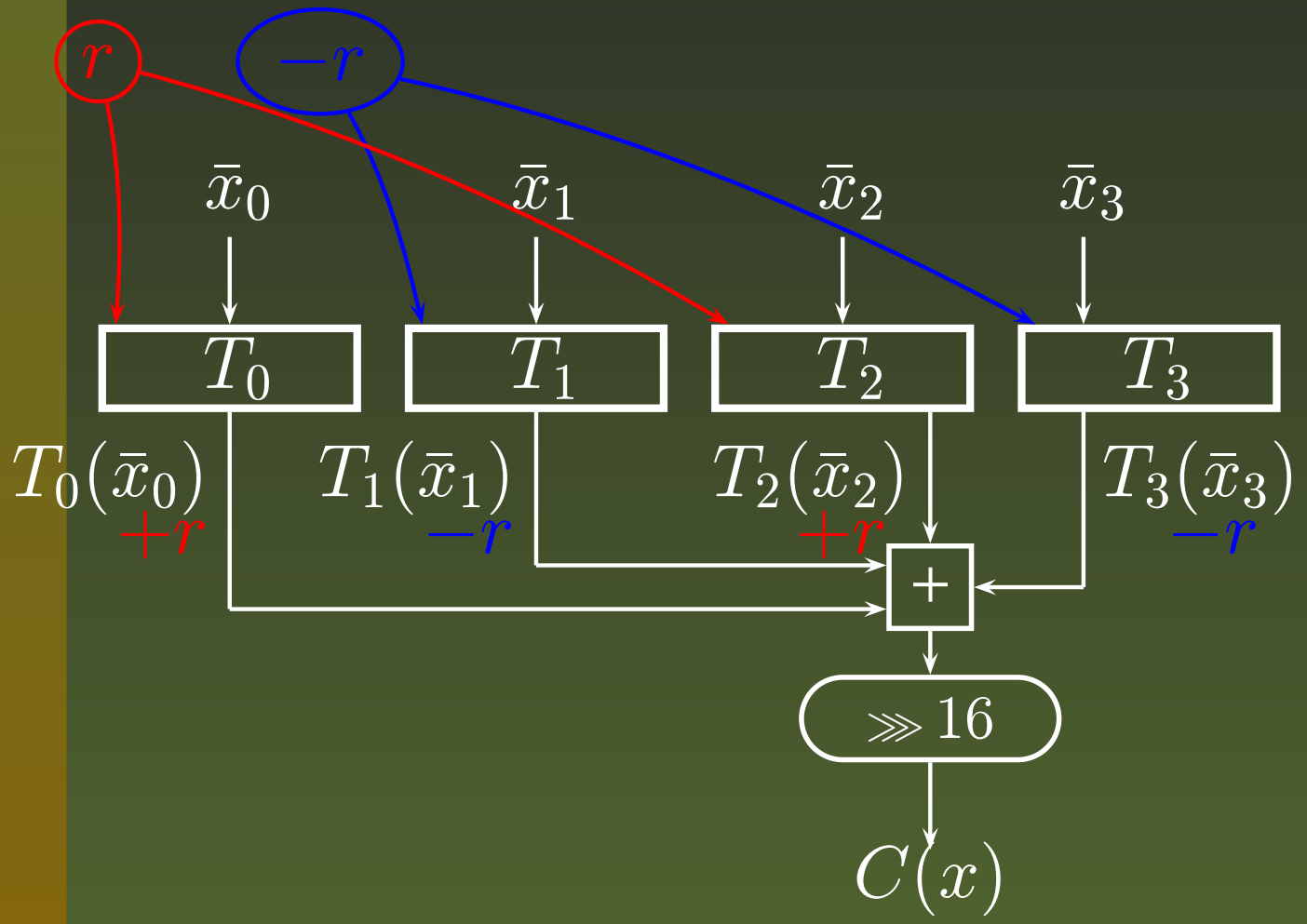


ABC: Function C, SCA

In applications subject to SCA we recommend to use masking:

- Modify each table by adding a random r or its additive inverse $-r$ to the table elements depending on the parity of the table number.

ABC: Function C, SCA



Properties of the ABC design pattern

Provable properties of the ABC key stream:

- The period of $(2^{127} - 1) \cdot 2^{32}$ words;
- Uniformly distributed key stream: $\forall$ 32-bit word a the number $\mu(a)$ of occurrences of a on the period satisfies:

$$\left| \frac{\mu(a)}{(2^{127} - 1) \cdot 2^{32}} - \frac{1}{2^{32}} \right| < \frac{1}{\sqrt{(2^{127} - 1) \cdot 2^{32}}};$$

- High linear complexity λ of the key stream: $2^{31} \cdot (2^{127} - 1) + 1 \geq \lambda \geq 2^{31} + 1$.

Properties of ABC circuit: Notes

- As a matter of fact we have proved the group of statements for a larger class of A, B, C. Thus, the designer can choose the maps suitable for the specific requirements.
- Note that the fact that these crucial security properties are proven does not exclude the necessity to analyse the concrete representations of A, B and C with respect to the whole set of cryptographical attacks.

ABC: Key dependence, State space

The following values can be (almost) freely defined without worsening the security properties of the resulting ABC mapping:

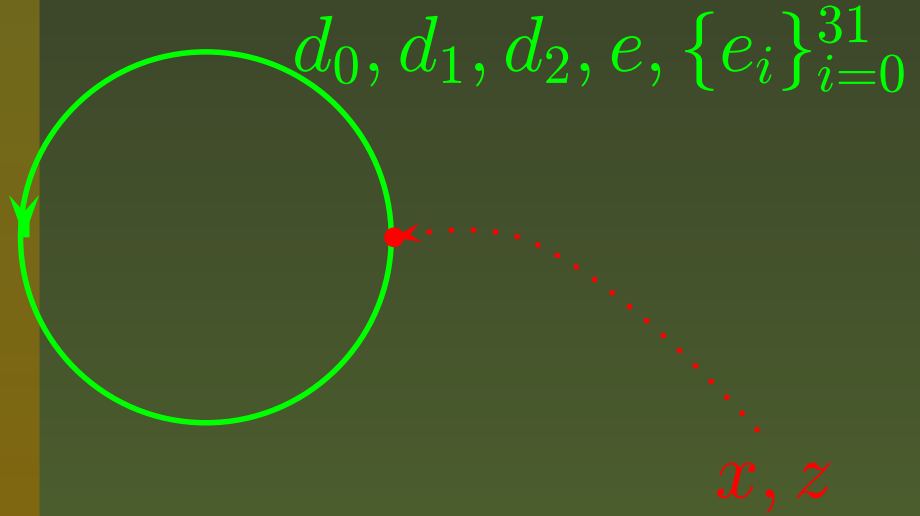
- A: The initial state $z \in (\mathbb{Z}/2^{32}\mathbb{Z})^4$;
- B: The coefficients $d_0, d_1, d_2 \in \mathbb{Z}/2^{32}\mathbb{Z}$ and initial state $x \in \mathbb{Z}/2^{32}\mathbb{Z}$;
- C: The coefficients $e, e_0, \ldots, e_{31} \in \mathbb{Z}/2^{32}\mathbb{Z}$.

NB! All up to restrictions imposed above!
Altogether we have about **1290 bits** to be freely set. Note that not all the bits have the same impact on the security of the cipher.

ABC: Key dependence, Cycles

The ABC stream cipher defines a family of cycles of length $2^{32}(2^{127}-1)$ words in the following way:

- $d_0, d_1, d_2, e, e_0, \ldots, e_{31}$ define a concrete cycle of length $P = 2^{32}(2^{127}-1)$;



- x, z select a start point on the cycle defined (exactly $2^{32}(2^{127}-1)$ variants).

ABC: Speed & Memory consumption

- A ***generic* reference C** implementation on a standard 3.2 GHz Intel Pentium 4 processor under Linux.
- Minimum **132** byte memory used.

w	Speed, Gbps	Cycles per byte	Table memory, bytes
2	2.25	11.38	256
4	4.24	6.04	512
8	**6.86**	3.73	4096

ABC: Conclusion

- Freedom to choose mappings A, B, C;
- Important security properties are ***proven***;
- Novel approach to ***counter-dependence***;
- High degree of ***key-dependence***;
- ***Key*** material usage ***flexibility***;
- High ***flexibility*** in terms of ***memory consumption***;
- Extremely high throughput rate of a ***generic*** ANSI C implementation - **6.9 Gbps, or 3.7 clocks/byte** on a Pentium 4 processor.